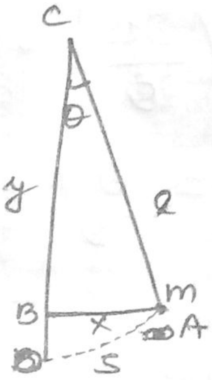
SIMPLE PENDULUM :-

let θ be the angular displacement of the simple pendulum from the equilibrium position. If 'l' be the effective length of the pendulum and 'm' be the mass of the bob then the displacement along arc $OA = S$

$$S = l\theta \left[\text{angle} = \frac{\text{arc}}{\text{Radius}} \right]$$



Kinetic energy (T) =

$\cancel{T = \frac{1}{2}mv^2 (T = \frac{1}{2}}$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$$

$$= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \quad \text{— (1)}$$

Where $\sin\theta = \frac{x}{l}$, $x = l\sin\theta$

& $\cos\theta = \frac{y}{l}$, $y = l\cos\theta$

So, $\dot{x} = l\cos\theta\,\dot{\theta}$

$\dot{y} = -l\sin\theta\,\dot{\theta}$

putting these values in eqn (1)

$$T = \frac{1}{2}ml^2\dot{\theta}^2$$

and potential energy at C

$$V = -mgy$$

$$= -mgl\cos\theta$$

Then $L = T - V = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$

From, Langrange's equation for Conservative System,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0.$$

i.e, $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0. \quad \text{——— (2)}$$

Now, $$\frac{\partial L}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{1}{2} m l^2 \dot{\theta}^2 + mgl\cos\theta\right)$$

$$= -mgl\sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta}$$

Putting these values in eqn (2)

$$\frac{d}{dt}\left(ml^2\dot{\theta}\right) + mgl\sin\theta = 0$$

or, $$ml^2\ddot{\theta} + mgl\sin\theta = 0$$

or, $$\boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0} \quad \text{——— (3)}$$

This represents the equation of motion of a simple pendulum.

For, small amplitude oscillation,

$\sin\theta \simeq \theta$, then, from eqn (3)

$$\boxed{\ddot{\theta} + \frac{g}{l}\theta = 0}$$ & Time period $T = 2\pi\sqrt{l/g}$

=x=x=

Teacher's Signature ________